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# A Tangential Guidance Logic for Virtual Target Based Path Following

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**A new guidance logic for UAV path following using a virtual target is presented. With respect to the line-of-sight, the UAV maintains an equal and opposite lead angle as that of the virtual target. A linear analysis of the resulting error dynamics presents a faster convergence to straight line path as compared to existing methods. With no overshoot in the response, the method presents a smooth following trajectory. Comparative simulations are carried out complying with the analytic findings.**

## Nomenclature

| | | |
|---|---|---|
| $A$ | = | UAV Position |
| $T$ | = | Virtual Target Position |
| $V_a$ | = | Velocity of the UAV |
| $\alpha_a$ | = | Heading angle of the Vehicle |
| $a_a$ | = | Lateral acceleration of the vehicle |
| $\theta$ | = | Line-of-Sight angle |
| $R$ | = | Line-of-Sight distance |
| $R^*$ | = | Constant Line-of-Sight Distance |
| $V_t$ | = | Velocity of the virtual target |
| $\alpha_t$ | = | Heading angle of the virtual target |
| $\sigma$ | = | Angle between Tangent and Secant to the circle |

## I. Introduction

Guiding itself to a virtual target moving at a fixed distance ahead on a desired path, an unmanned aerial vehicle (UAV) can eventually converge to the path. The use of pure pursuit guidance (heading directly towards the virtual target) is widely reported for following the virtual target.[1–4] Since pure pursuit ends in a tail chase with respect to the virtual target, the UAV converges to the desired path following the virtual target. While this works well in the case of straight line paths, pure pursuit performance deteriorates drastically while following a virtual target on curved paths. Pointing towards the virtual target a distance ahead, the vehicle cuts large corners on curved sections of the path. Line-of-sight guidance based virtual target following is described in Refs.[5–7] In,[8] proportional navigation is used for virtual target based path following. Line-of-sight guidance requires a third reference point for its implementation and proportional navigation terminal heading is very sensitive to the choice of navigation gain.

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A popular nonlinear guidance logic for path following was proposed by Park et al.,[9–11] wherein the lateral acceleration is dependent on the distance to the virtual target and the heading error with respect to it. The UAV follows an instantaneous radius of curvature which increases with reducing cross-track errors finally leading to convergence. Though the logic leads to smooth convergence for various initial conditions, it does not take into account the virtual target heading or the path direction for generating guidance commands. Further, the resulting error dynamics is of second order leading to a limitation on the choice virtual target locations along a curved path.

The use of trajectory shaping guidance law,[12] another impact angle control method for missiles, as a prospective path following logic was discussed by Ratnoo et al.[13] Therein, the lateral acceleration depends on the instantaneous line-of-sight angle (orientation of the line joining the vehicle and virtual target), headings of the UAV and the virtual target and the distance to virtual target. The resulting guidance command, under small angle assumptions, generates the same instantaneous radius of curvature as that of the virtual target following a circular path. However, implementing the logic requires time-to-go computations with respect to the virtual target, approximations for which can lead to errors in path following.

This work revisits the problem as presented in Refs. 9–11 and proposes a logic which now incorporates the virtual target heading direction. The UAV follows a circular arc which is tangential to the path at the virtual target position. In other words, the heading of the virtual target and that UAV are now tangents to the same circular arc. This results in a faster transition of the UAV towards the desired path, and generates maneuvers based on the virtual target heading.

The remainder of paper is arranged as follows: Section II describes the virtual target based path following problem. The proposed tangential guidance logic is presented in Section III. Section IV presents the linear analysis of the proposed guidance law. Comparison with existing methods is presented in Section V. Section VI presents simulation studies. Concluding remarks are presented in Section VII.

## II. Virtual Target Based Path Following

Consider the scenario shown in Fig 1 wherein a UAV tries to follow a desired path by considering a virtual target moving on the path. The UAV moves with the speed $V_a$. Along the path, the virtual target moves at a fixed distance $R^*$ ahead of the UAV. The resulting virtual target speed is denoted by $V_t$. The objective here is to formulate prospective guidance laws which the UAV uses to follow the virtual target. Performance of such guidance laws can be measured in terms of the errors in following the path and the time taken to reach the path.

## III. The Proposed Guidance Law

Consider the detailed engagement geometry as shown in Fig. 2. Here, $\alpha_a$, $\alpha_t$, and $\theta$ represent UAV heading, virtual target heading, and line-of-sight angle as measure with respect to a fixed reference. The UAV moves with a constant speed and changes its heading direction using the lateral acceleration command $a_a$ as

$$\dot{\alpha_a} = \frac{a_a}{V_a} \tag{1}$$

The lead angles are the headings with respect to the line-of-sight angle $\theta$. According to the proposed logic the UAV maintains an equal and opposite lead angle as that of the target In terms of angular orientation the proposed logic can be expressed as,

$$\theta - \alpha_a = \alpha_t - \theta \tag{2}$$

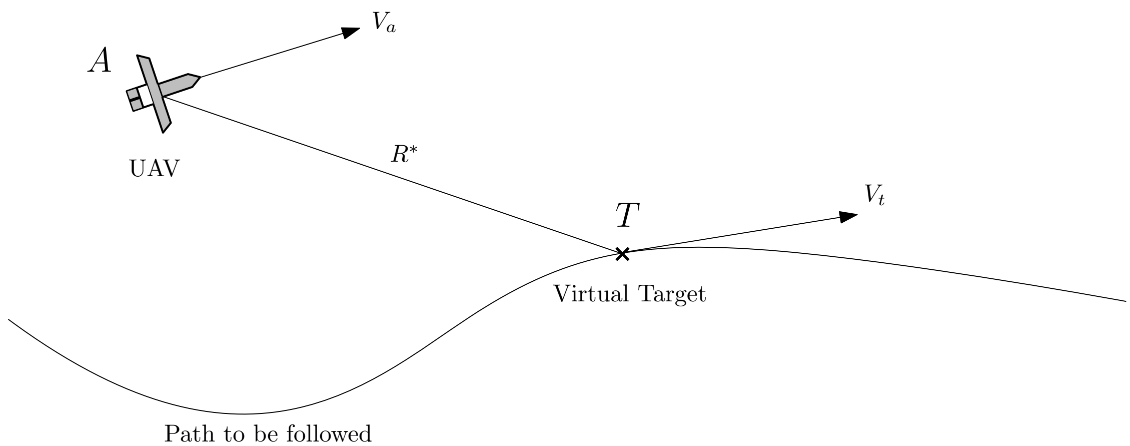


Figure 1: Virtual target based path following

leading to the relationship

$$\alpha_a = 2\theta - \alpha_t \tag{3}$$

which can be expressed as a desired UAV heading as governed by,

$$\alpha_d = 2\theta - \alpha_t \tag{4}$$

Note that the desired UAV heading is a function of the line-of-sight angle and target heading. Fig. 3 shows the UAV moving along the desired direction. Here, $\sigma$ denotes the magnitude of the lead angle. Note that, by following an equal and opposite lead angle, the UAV follows a tangential direction to the circle which passes through the instantaneous UAV and target positions and has target heading as a tangent.

As the UAV moves closer to the path, $\sigma$ reduces as can be seen in Fig. 4, finally leading the the UAV to converge to the path with $\sigma \to 0$. In the process, the instantaneous radius of curvature increases and tends to infinity. A detailed analysis of the error dynamics is presented in Section IV. From Fig 4,

$$R_a = \frac{R^*}{2\sin\sigma} \tag{5}$$

where, $R_a$ is the instantaneous radius of curvature. Fig. 5 presents a schematic of the evolving engagement geometry. As the UAV moves through the points $A_1,\ A_2,\ A_3$ the virtual target moves through $T_1,\ T_2,\ T_3$, respectively, maintaining constant separation. Note that, following the proposed tangential logic, the instantaneous circles $C_1,\ C_2,\ C_3$ increases in radii finally leading the UAV onto the path.

## IV. Linear Analysis of the Guidance Law for a Straight Line Path

In this section the convergence of the error or the lateral displacement from the desired path is discussed. A linear analysis is carried out considering straight line paths. Consider the scenario as shown in Fig. 6 wherein the UAV uses the proposed guidance logic of (4) to follow the straight line path. The reference is chosen with its $X$ axis along the desired path and hence $\alpha_T = 0$. The resulting desired heading of (4), in this case, simplifies to

$$\alpha_d = 2\theta \tag{6}$$

Assuming small angle $\theta$ and using (6), the position error rate can be expressed as

$$\dot{d} = V_a \sin(\alpha_d) = V_a \sin(2\theta) \approx 2V_a\theta \tag{7}$$

From Fig. 6

$$\theta = \sigma \approx -\frac{d}{R^*} \tag{8}$$

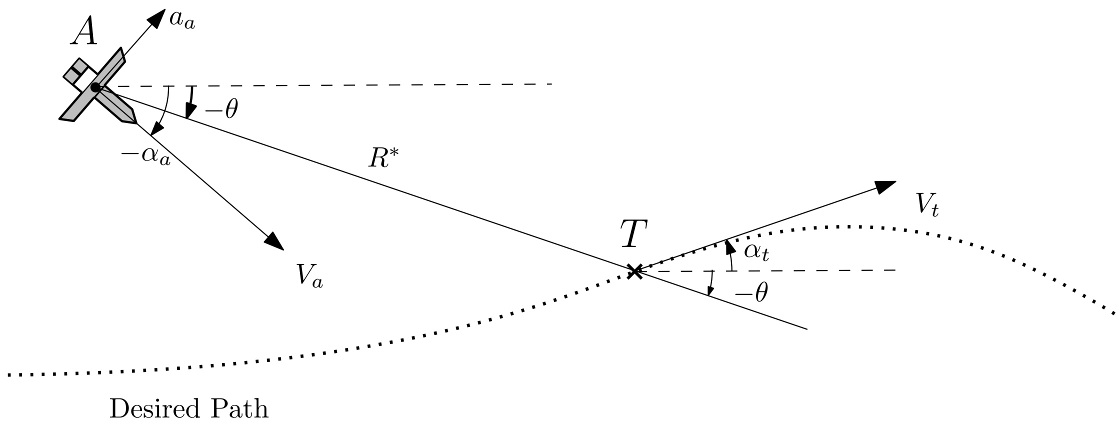


Figure 2: Detailed engagement geometry

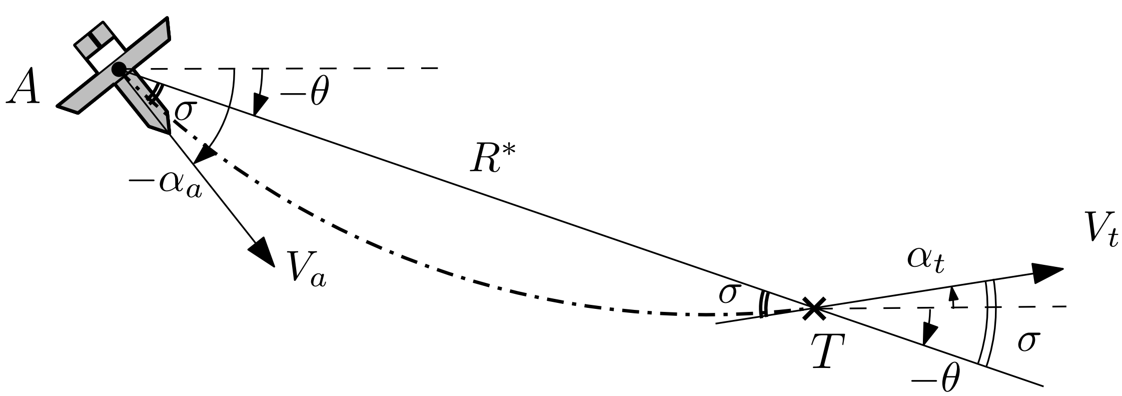


Figure 3: Schematic of the guidance logic

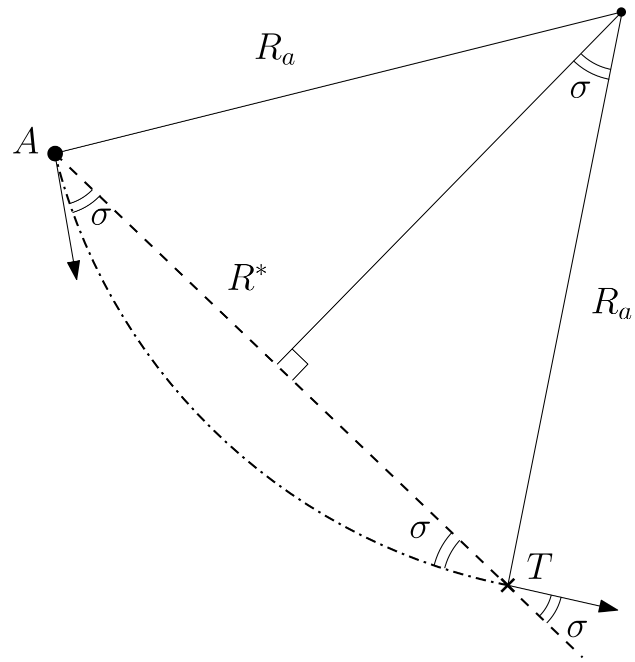


Figure 4: Instantaneous radius of curvature $R_a$

Using (8) in (7),

$$\dot{d} \approx -2\frac{V_a}{R^*}d \tag{9}$$

From (9), the error dynamics is of first order with the time constant $\frac{2V_a}{R^*}$.

## V. Comparisons with Existing Path Following Guidance Techniques

The settling characteristics of the tangential guidance logic is compared with the non-linear guidance logic[9], and the trajectory shaping based guidance [13]. The comparison is based on the performance with respect to the settling time $t_s$ (2%) for the lateral displacement.

### A. Nonlinear guidance logic [9]

The lateral acceleration for the Nonlinear guidance logic is given as

$$a_a = \frac{2V_a^2 \sin(\alpha_a - \theta)}{R^*} \tag{10}$$

where, $R^*$ is the constant distance to the virtual target which moves along the desired path in a way so as to maintain a constant $R^*$. The linear approximation of the dynamics of the Nonlinear guidance for following a straight line or a circular path results in the following second order equation.

$$\ddot{d} + \frac{2V_a}{R^*}\dot{d} + \left(\frac{V_a}{R^*}\right)^2 d \approx 0 \tag{11}$$

This results in the rate of convergence $\zeta\omega_n = \frac{V_a}{R^*}$. The settling time $t_s$ (2%) for the Nonlinear logic is given by

$$t_s = \frac{4R^*}{V_a} \tag{12}$$

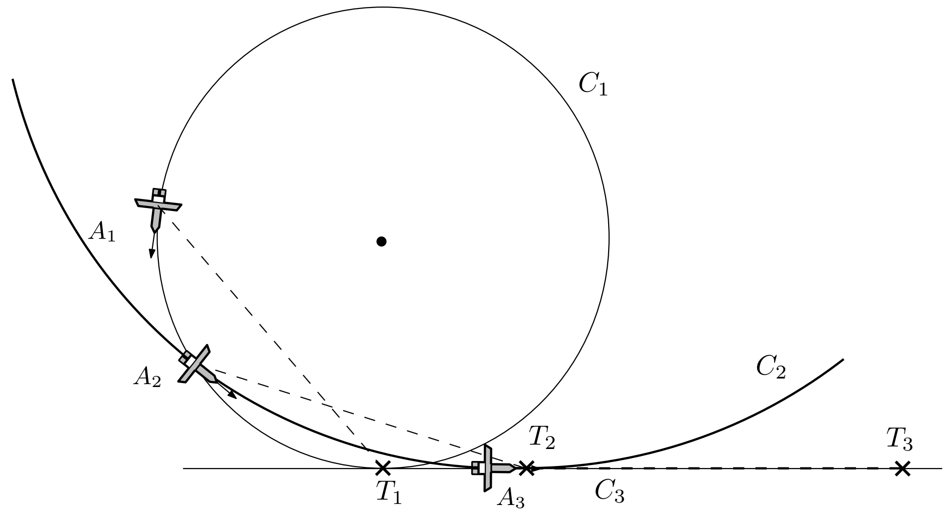


Figure 5: Evolving engagement geometry

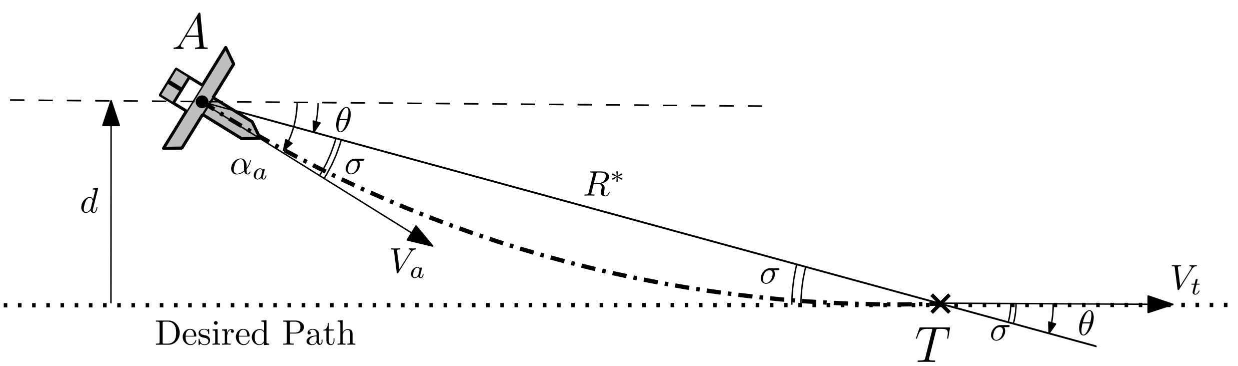


Figure 6: Straight line path

whereas, the settling time for the Tangential guidance logic can be derived by solving (9) fpr $R^* = R$ as follows

$$t_s = \frac{1.956R^*}{V_a} \tag{13}$$

As it can be observed from Eq. 12 and Eq. 13, the settling time for the tangential guidance logic is less than half of that of the Nonlinear guidance logic.

## B. Trajectory shaping guidance [13]

The expression for the lateral acceleration in trajectory shaping guidance is given by

$$a_a = \frac{V_a}{t_{go}}(-6\theta + 4\alpha_a + 2\alpha_t) \tag{14}$$

where $t_{go}$ is the time for an hypothetical interception. $t_{go}$ is approximated as $\frac{R^*}{V_a}$. The linear approximation of the dynamics of the trajectory shaping guidance for straight line and circular path is given by

$$\ddot{d} + 4\frac{V_a}{R^*}\dot{d} + 6\left(\frac{V_a}{R^*}\right)^2 d \approx 0 \tag{15}$$

The rate of convergence $\zeta\omega_n$ is $\frac{2V_a}{R^*}$, resulting in the $t_s$ (2%) settling time as,

$$t_s = \frac{2R^*}{V_a} \tag{16}$$

The $t_s$ (2%) for the tangential guidance is given by Eq. 13. Trajectory shaping guidance has a slightly higher settling time as compared to the tangential guidance logic. Although the difference is small, the tangential guidance follows a first order error dynamics whereas trajectory shaping follows a second order error dynamics. This results in an overshoot for the trajectory shaping. This aspect may result in discrepancies while following a curved path with regards to the choice of $R^*$. Also, the calculation of $t_{go}$ is necessary for trajectory shaping guidance which is approximated as $\frac{R^*}{V_a}$ and may not be accurate.

# VI. Simulations

Simulations are carried out for $V_a = 15$ m/s moving in a plane. For implementing the tangential guidance law the lateral acceleration guidance command is used at any initial heading angle, an error term is added to the lateral acceleration. The lateral acceleration is given by Eq. (17).

$$a_a = V_a\dot{\theta} - K(\alpha_d - \alpha_a) \tag{17}$$

where K is a constant and

$$\alpha_d = 2\theta - \alpha_t \tag{18}$$

The first term is the lateral acceleration command of Eq. (17) tracks the rotation of the line-of-sight and the second term annuls the errors with respect to the desired heading. Comparative studies are carried out with respect to the nonlinear guidance logic [9] and trajectory shaping guidance [13].

## A. Straight line following

Fig. 7 shows the case where the three guidance laws are simulated to follow a straight line path. The initial heading in this case is aligned as per Eq. (2). The tangential guidance logic and trajectory shaping guidance generate similar trajectories as shown in Fig. 7a. Nonlinear guidance leads to a much slower convergence to the path. The angular variations are plotted in Fig. 7b and the corresponding lateral acceleration profiles are plotted in Fig. 7c. The 2% settling time $t_s$ values of the error for the tangential guidance logic, trajectory shaping and the nonlinear guidance are 5.362 s, 5.440 s and 12.01 s respectively. Fig. 7d plots the position error variation with time.

## B. Circular path following

A sample simulation scenario for circular path following is shown in Fig. 8. Similar trends are observed for the settling time comparison. With no overshoot, the tangential guidance trajectory converges to the circle externally. The angular variations and lateral acceleration profiles are plotted in Fig. 8b and Fig. 8c, respectively. Fig. 8d plots the position error variation.

# VII. Conclusion

With equal and opposite lead angle as that of the virtual target a new guidance logic is presented for UAV path following. Linear analysis of the resulting engagement leads to a first order dynamics and closed-form expressions are obtained for the settling time considering a straight line path. Comparison with the existing circular maneuver logic highlights a twice as fast convergence to the path. Sample simulations are carried out in support of the proposed method.

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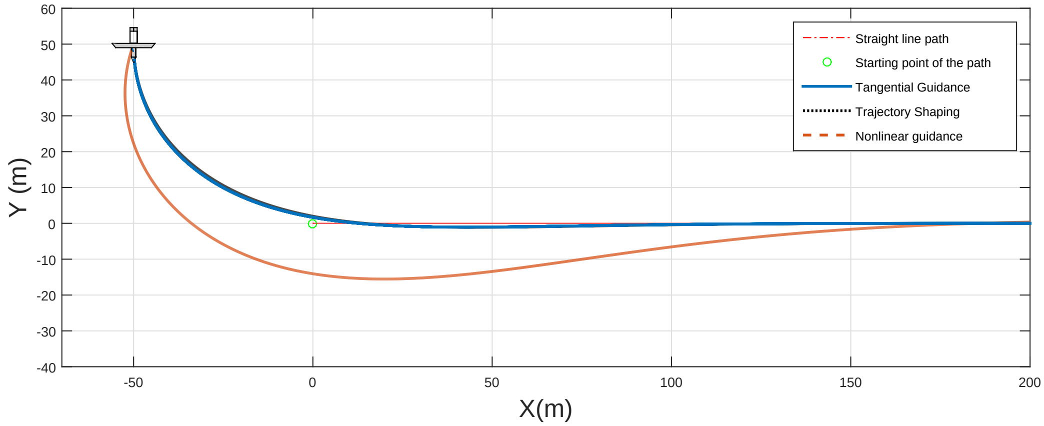
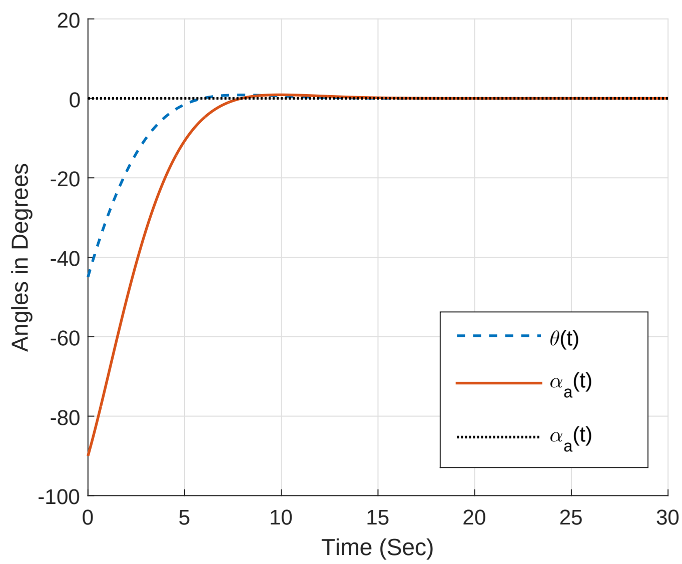
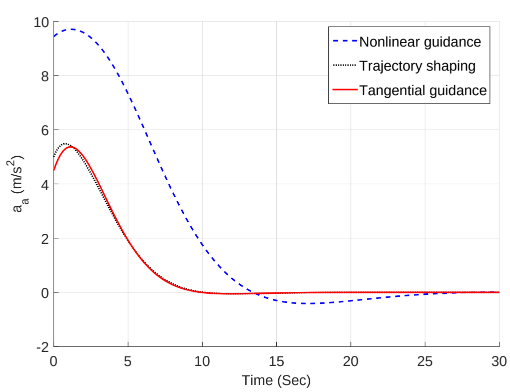


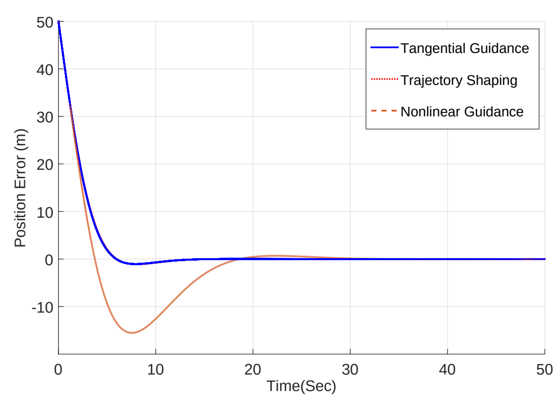

(a) Trajectories




(b) Variation of $\theta$, $\alpha_a$ and $\alpha_t$ with time

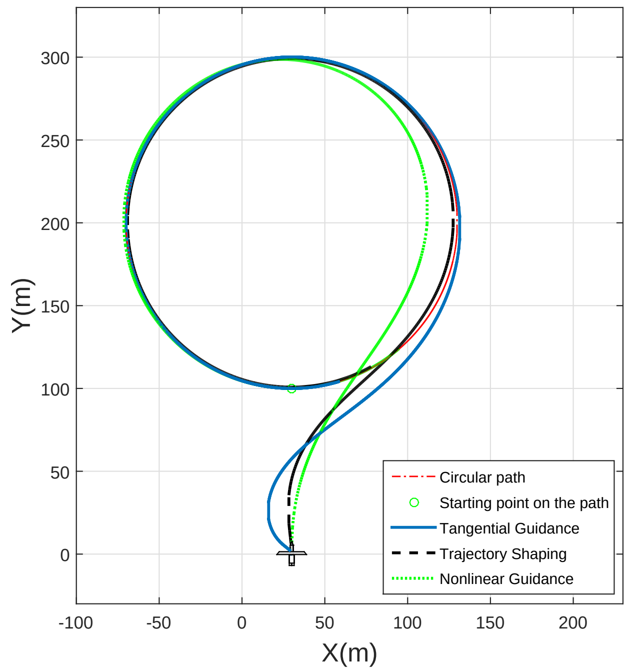



(c) Lateral acceleration profile

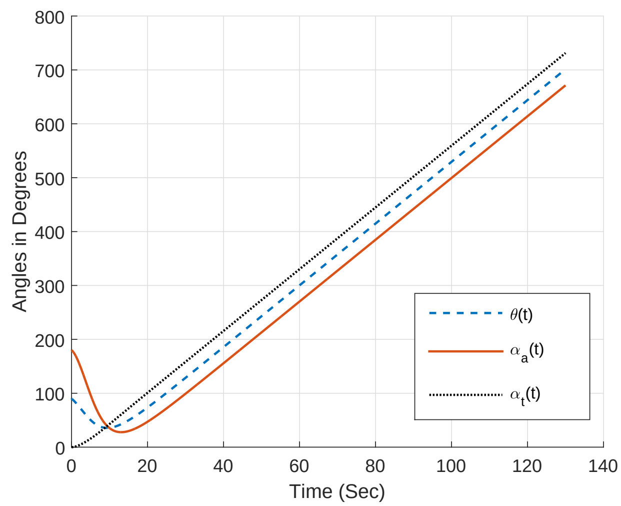



(d) Position error with time

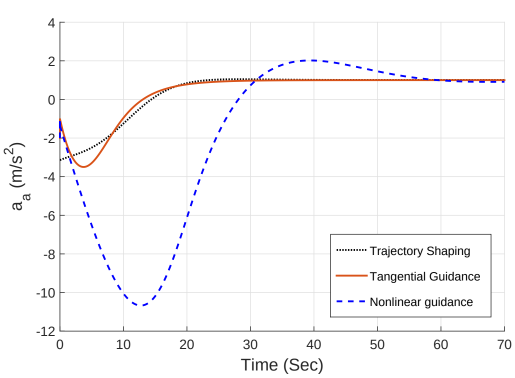
Figure 7: Straight line path following



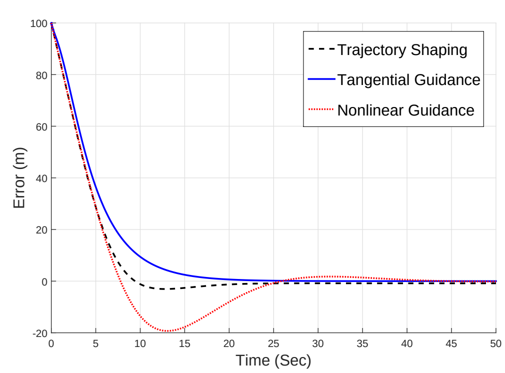

(a) Trajectories




(b) Variation of $\theta$, $\alpha_a$ and $\alpha_t$ with time




(c) Lateral acceleration profile




(d) Position error with time

Figure 8: Circular path following

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